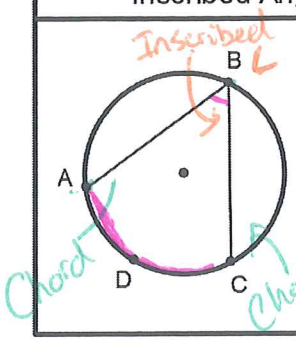
Geometry - 10.4 - Inscribed Angles

Inscribed Angle Theorem (10.5)



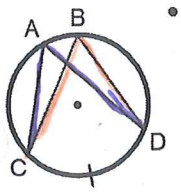
- An Inscribed angle is half the measure of its intercepted arc.

$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

or

$2 \cdot m\angle ABC = m\overset{\frown}{AC}$

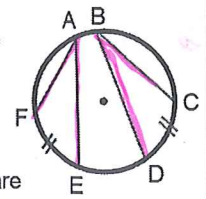
Theorem 10.6



- If two inscribed angles of a circle intercept Congruent arcs or the Same arc, then the angles are Congruent

$\angle A \cong \angle B$

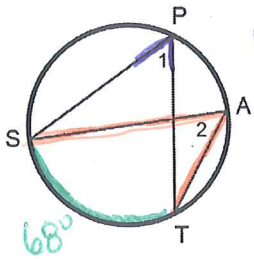
Share $\overset{\frown}{CD}$



$\angle A \cong \angle B$

$\overset{\frown}{FE} \cong \overset{\frown}{ED}$

Ex 1 - If $m\overset{\frown}{ST} = 68°$, find $m\angle 1$ and $m\angle 2$.



$m\angle 1 = \frac{1}{2} m\overset{\frown}{ST}$

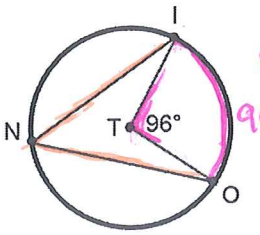
$m\angle 1 = \frac{1}{2} \cdot 68°$

$= 34°$

$m\angle 1 \cong m\angle 2$

$m\angle 2 = 34°$

Ex 2 - Given T is the center of the circle, find $m\overset{\frown}{IO}$ and $m\angle INO$.

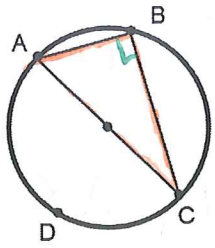


$m\overset{\frown}{IO} = 96°$

$\angle N = \frac{1}{2} \cdot 96$

$= 48°$

Theorem 10.7



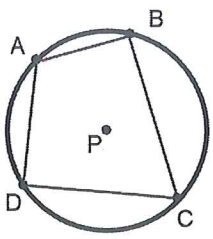
- If an inscribed angle intercepts a semicircle, then the angle is a right angle.

$\overline{AC}$ is a diameter

↓

$\angle B$ is right

Theorem 10.8



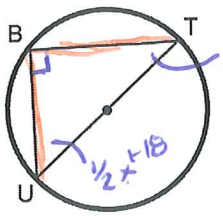
- If a quadrilateral is Inscribed in a circle, then its opposite angles are Supplementary

If Inscribed quad

↓

$m\angle A + m\angle C = 180 = m\angle B + m\angle D$

Ex 3 - If $m\angle BUT = \frac{1}{2}x + 18$ and $m\angle BTU = \frac{2}{5}x$, find the measure of all three angles in the triangle.



$10\left(\frac{1}{2}x + 18 + \frac{2}{5}x = 90\right)$

$5x + 180 + 4x = 900$

$9x + 180 = 900$

$-180 \quad -180$

$\frac{9x}{9} = \frac{720}{9}$

$x = 80$

$\angle B = 9$

$\angle U = \frac{1}{2}x + 18$

$\frac{1}{2}(80) + 18$

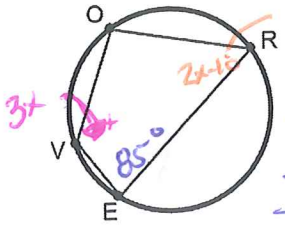
$40 + 18$

58

$\angle T = \frac{2}{5}(80)$

$= 32°$

Ex 4 - If $m\angle E = 85°$, $m\angle R = (2x - 10)°$, and $m\angle V = (3x)°$, find $m\angle O$, $m\angle R$, and $m\angle V$.



$m\angle E + m\angle O = 180°$

$85° + m\angle O = 180$

$-85° \quad -85$

$m\angle O = 95$

$2(38) - 10$

$66°$

$m\angle V + m\angle R = 180$

$3x + 2x - 10 = 180$

$5x - 10 = 180$

$+10 \quad +10$

$5x = 190$

$\frac{5x}{5} = \frac{190}{5}$

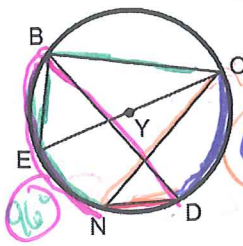
$x = 38$

$m\angle V = 3x$

$= 3(38)$

$= 114$

Ex 5 - In $\odot Y$, $\overline{EO}$ is a diameter, $m\overset{\frown}{OD} = 68°$ and $m\overset{\frown}{BN} = 96°$. Find $m\angle EBO$, $m\angle BDN$, and $m\angle OND$.



$68°$

$96°$

$\angle EBO = 90°$

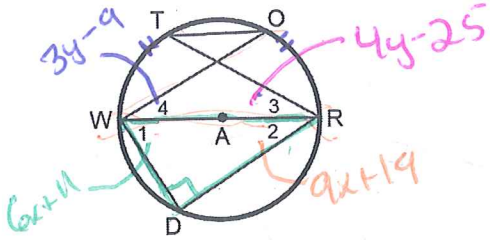
$m\angle BDN = \frac{1}{2} \cdot 96$

$= 48°$

$m\angle OND = 68 \cdot \frac{1}{2}$

$= 34°$

Ex 6 - In $\odot A$, $m\angle 1 = (6x + 11)°$, $m\angle 2 = (9x + 19)°$, $m\angle 3 = (4y - 25)°$, $m\angle 4 = (3y - 9)°$, and $\overset{\frown}{WT} \cong \overset{\frown}{RO}$. Find the measures of angles 1 through 4.



$6x + 11 + 9x + 19 = 90°$

$15x + 30 = 90°$

$15x = 60$

$x = 4$

$3y - 9 = 4y - 25$

$-3y + 25 \quad -3y + 25$

$16 = y$

$m\angle 1 = 6x + 11$

$= 6(4) + 11$

$24 + 11$

35

$m\angle 2 = 9x + 19$

$= 9(4) + 19$

$36 + 19$

$55°$

$m\angle 3 = 4y - 25$

$4(16) - 25$

$64 - 25$

$39°$

$m\angle 4 = 3y - 9$

$= 3(16) - 9$

$48 - 9$

$39°$