Geometry - 2.2 - Logic

- A Statement is any sentence that is either True or False, but not both. The truth or falsity of a statement is called its truth value.
- Statements are often represented with the letters p and q.
- The negation of a statement has the opposite meaning as well as the opposite truth value. The negation of a statement *p* is not p, or, in symbols, ~p.
- Two or more statements can be joined together to form a Compound statement.
- A Conjunction is a compound statement formed by joining two or more statements with the word and. The conjuction of two statements *p* and *q* is p and q, or, in symbols, $p \wedge q$.
- A Disjunction is a compound statement formed by joining two or more statements with the word or. The disjuction of two statements *p* and *q* is p or q, or, in symbols, $p \vee q$.

- A conjunction is only true when all of its statements are true.

Ex 1 - Use the following statements to write a compound statement for each conjunction. Then find its truth value.

T *p*: A heptagon has 7 sides.
F *q*: -18 + (-17) = 35 → (-35)
T *r*: Ottawa is the capital of Canada.

a) *p* and *q* = a heptagon has 7 sides, and -18 + (-17) = 35
($p \wedge q$) False → Both must be true with conjunctions

b) $r \wedge p$ = Ottawa is the capital of Canada, and a heptagon has 7 sides.
(r and p)
(T) (T) True → Both are true, so it is a conjunction

c) *p* and not *q*
($p \wedge \sim q$) = a heptagon has 7 sides, and -18 + (-17) ≠ 35
(T) (T) True

d) $\sim r \wedge p$ = Ottawa is NOT the capital of Canada, and a heptagon has 7 sides.
(not r and p)
F T √ False

- A disjunction is true when any of its statements are true.

Ex 2 - Use the following statements to write a compound statement for each disjunction. Then find its truth value.

(F) p: $-19 \geq 18$ $-19 \leq 18$
(T) q: An equilateral triangle has 3 congruent sides.
(F) r: Acapulco is the capital of Mexico. → Mexico City

a) p or q = $-19 \geq 18$ OR an equilateral triangle has 3 congruent sides

$(p \vee q)$
F "OR" T
True → only one statement needs to be true in disjunctions

b) $p \vee r$ = $-19 \geq 18$, or Acapulco is the capital of Mexico.

(p or r)
(F) (F)
False → No statement is true, disjunction is false

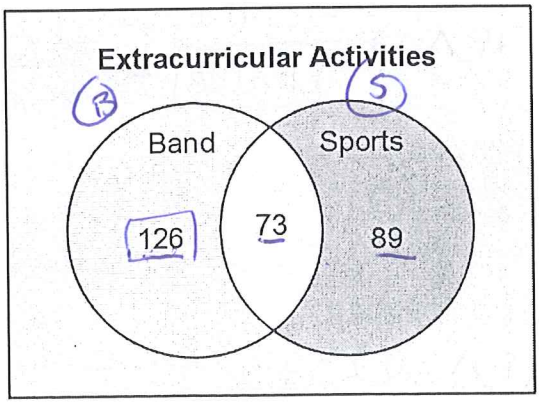
d) $\sim p \vee \sim r$ = $-19 \not\geq 18$ or Acapulco is not the capital of Mexico

(Not p or not r)
(T) (T)

True

- Conjunctions and disjunctions can both be illustrated with Venn diagrams. For two sets A and B, a conjunction is the intersection of A and B (written $A \cap B$) while a disjunction is the union of A and B (written $A \cup B$).

Ex 3 - The Venn diagram shows the number of students in a school that play in band and play on a sports team.



a) How many students play both in band and on a sports team?

$(B \cap S) = 73$ students

b) How many students play in band or on a sports team?

$(B \cup S) = 126 + 73 + 89 = 288$ students

c) How many students play in band but not on a sports team?

$(B \cap \sim S) = 126$ students

- A Truth tables is a convenient way to organize truth values of statements.

Ex 3 - Construct a truth table for each compound statement.

a) $\sim p \vee q$ (OR)

Step ① →

p	q	~p	~p ∨ q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b) $\sim p \wedge \sim q$ (← and)

p	q	~p	~q	~p ∧ ~q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

c) $(p \vee q) \wedge r$

p	q	p ∨ q	r	(p ∨ q) ∧ r
T	T	T	T	T
T	T	T	F	F
T	F	T	T	T
T	F	T	F	F
F	T	T	T	T
F	T	T	F	F
F	F	F	T	F
F	F	F	F	F

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