2-2 Study Guide and Intervention

Logic

Determine Truth Values A **statement** is any sentence that is either true or false. The truth or falsity of a statement is its **truth value**. A statement can be represented by using a letter. For example,

Statement p: Chicago is a city in Illinois. The truth value of statement p is true.

Several statements can be joined in a **compound statement**.

Statement p and statement q joined by the word *and* is a **conjunction**.	Statement p and statement q joined by the word *or* is a **disjunction**.	**Negation:** *not p* is the negation of the statement p.
Symbols: $p \wedge q$ (Read: *p and q*)	Symbols: $p \vee q$ (Read: *p or q*)	Symbols: $\sim p$ (Read: *not p*)
The conjunction $p \wedge q$ is true only when both p and q are true.	The disjunction $p \vee q$ is true if p is true, if q is true, or if both are true.	The statements p and $\sim p$ have opposite truth values.

Example 1 Write a compound statement for each conjunction. Then find its truth value.

p: An elephant is a mammal.
q: A square has four right angles.

a. $p \wedge q$

Join the statements with *and*: An elephant is a mammal and a square has four right angles. Both parts of the statement are true so the compound statement is true.

b. $\sim p \wedge q$

$\sim p$ is the statement "An elephant is not a mammal." Join $\sim p$ and q with the word *and*: An elephant is not a mammal and a square has four right angles. The first part of the compound statement, $\sim p$, is false. Therefore the compound statement is false.

Example 2 Write a compound statement for each disjunction. Then find its truth value.

p: A diameter of a circle is twice the radius.
q: A rectangle has four equal sides.

a. $p \vee q$

Join the statements p and q with the word *or*: A diameter of a circle is twice the radius or a rectangle has four equal sides. The first part of the compound statement, p, is true, so the compound statement is true.

b. $\sim p \vee q$

Join $\sim p$ and q with the word *or*: A diameter of a circle is not twice the radius or a rectangle has four equal sides. Neither part of the disjunction is true, so the compound statement is false.

Exercises

Write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $10 + 8 = 18$ q: September has 30 days. r: A rectangle has four sides.

1. p and q

2. p or r

3. q or r

4. q and $\sim r$

2-2

Study Guide and Intervention (continued)

Logic

Truth Tables One way to organize the truth values of statements is in a **truth table**. The truth tables for negation, conjunction, and disjunction are shown at the right.

Negation	
p	**~p**
T	F
F	T

Conjunction		
p	**q**	**p ∧ q**
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction		
p	**q**	**p ∨ q**
T	T	T
T	F	T
F	T	T
F	F	F

Example 1 Construct a truth table for the compound statement *q* or *r*. Use the disjunction table.

q	r	q or r
T	T	T
T	F	T
F	T	T
F	F	F

Example 2 Construct a truth table for the compound statement *p* and (*q* or *r*).

Use the disjunction table for (*q* or *r*). Then use the conjunction table for *p* and (*q* or *r*).

p	q	r	q or r	p and (q or r)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Exercises

Contruct a truth table for each compound statement.

1. *p* or *r*

2. ~*p* ∨ *q*

3. *q* ∧ ~*r*

4. ~*p* ∧ ~*r*

5. (*p* and *r*) or *q*

2-2 Skills Practice

Logic

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $-3 - 2 = -5$

q: Vertical angles are congruent.

r: $2 + 8 > 10$

s: The sum of the measures of complementary angles is 90°.

1. p and q

2. $p \wedge r$

3. p or s

4. $r \vee s$

5. $p \wedge \sim q$

6. $q \vee \sim r$

Copy and complete each truth table.

7.

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$
T	T			
T	F			
F	T			
F	F			

8.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	
T	F	T	
F	T	F	
F	F	T	

Construct a truth table for each compound statement.

9. $\sim q \wedge r$

10. $\sim p \vee \sim r$

2-2 Practice

Logic

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: 60 seconds = 1 minute
q: Congruent supplementary angles each have a measure of 90.
r: $-12 + 11 < -1$

1. $p \wedge q$

2. $q \vee r$

3. $\sim p \vee q$

4. $\sim p \wedge \sim r$

Copy and complete each truth table.

5.

p	*q*	*~p*	*~q*	*~p* $\vee$ *~q*
T	T			
T	F			
F	T			
F	F			

6.

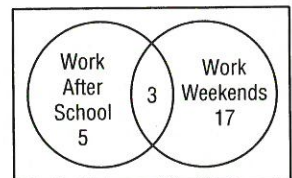| *p*	*q*	*~p*	*~p* $\vee$ *q*	*p* $\wedge$ (*~p* $\vee$ *q*)
T	T			
T	F			
F	T			
F	F			

Construct a truth table for each compound statement.

7. $q \vee (p \wedge \sim q)$

8. $\sim q \wedge (\sim p \vee q)$

SCHOOL For Exercises 9 and 10, use the following information.

The Venn diagram shows the number of students in the band who work after school or on the weekends.

9. How many students work after school and on weekends?

10. How many students work after school or on weekends?