Name: Key

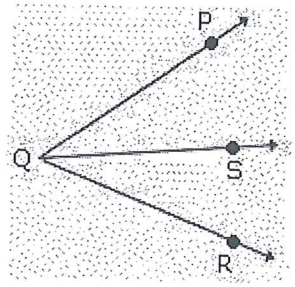
2-8 Proving Angle Relationships Notes

Objectives:

- Students will write proofs involving supplementary and complementary angles
- Students will write proofs involving congruent and right angles.

Postulate 2.11: Angle Addition Postulate:

- If S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.



Example 1: Using the Angle Addition Postulate:

If $m\angle ABC = 103$ find the value of x. Then find $m\angle ABD$ and $m\angle DBC$

$m\angle ABD + m\angle DBC = m\angle ABC$

$3x + 1 + 4x - 3 = 103$

$7x - 2 = 103$

$+2 \quad +2$

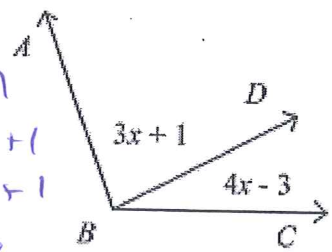
$7x = 105$

$\frac{7x}{7} = \frac{105}{7}$

$x = 15$

$m\angle ABD = 3x+1 = 3(15)+1 = 45+1 = 46$

$m\angle DBC = 4x-3 = 4(15)-3 = 60-3 = 57$



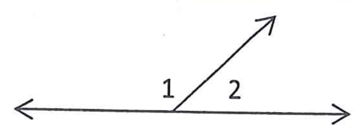
x= 15

$m\angle ABD =$ 46

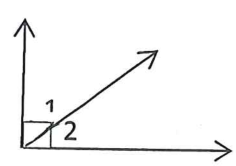
$m\angle DBC =$ 57

Supplementary and Complementary Angles

- Theorem 2.3: If two angles form a Linear Pair, then they are Supplementary angles.
- Theorem 2.4: If the non-common sides of two adjacent angles form a right angle, then the angles are complimentary angles.



$m\angle 1 + m\angle 2 = 180$



$m\angle 1 + m\angle 2 = 90$

Example 2: Using Supplementary Angles

If $\angle 1$ and $\angle 2$ form a linear pair, and $m\angle 1 = 4x - 5$ and the $m\angle 2 = 14x + 5$, find x and the measurements of $\angle 1$ and $\angle 2$.

$m\angle 1 + m\angle 2 = 180°$

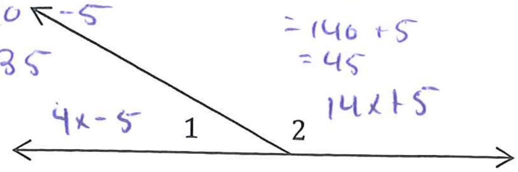
$4x - 5 + 14x + 5 = 180°$

$\frac{18x}{18} = \frac{180°}{18}$

$x = 10$

$m\angle 1 = 4x-5 = 4(10)-5 = 40-5 = 35$

$m\angle 2 = 14x+5 = 14(10)+5 = 140+5 = 45$



x= 10

$\angle 1 =$ 35°

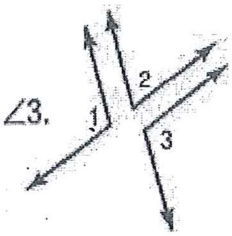
$\angle 2 =$ 45

Theorem 2.5	**Angle Congruence**
Reflexive Property	$\angle 1 \cong \angle 1$
Symmetric Property	**If $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 1$**
Transitive Property	**If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$**

Theorem 2.6: Angles supplementary to the same angle or to congruent angles are congruent.

If $m\angle 1 + m\angle 2 = 180$, and $m\angle 2 + m\angle 3 = 180$, then $\underline{\angle 1} \cong \underline{\angle 3}$.

If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



Example 3: Proof of Theorem 2.6

Given: $\angle 1$ and $\angle 2$ are supplementary

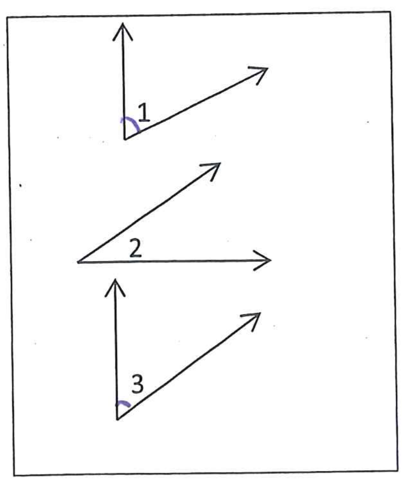
$\angle 2$ and $\angle 3$ are supplementary

Prove: $\angle 1 \cong \angle 3$

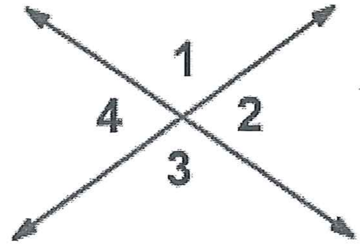
Statements	**Reasons**
1. $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary	1. Given
2. $m\angle 1 + m\angle 2 = 180$ $m\angle 2 + m\angle 3 = 180$	2. Definition of supplementary $\angle$'s
3. $m\angle 1 + m\cancel{\angle 2} = m\cancel{\angle 2} + m\angle 3$ $-m\angle 2$ $\quad -m\angle 2$	3. Substitution.
4. $m\angle 1 = m\angle 3$	4. subtraction
5. $\angle 1 \cong \angle 3$	5. Definition of $\cong$ $\angle$'s

Theorem 2.7: Angles complementary to the same angle or to congruent angles are congruent.

If $m\angle 1 + m\angle 2 = 90$, and $m\angle 2 + m\angle 3 = 90$, then $\underline{\angle 1} \cong \underline{\angle 3}$.



Theorem 2.8: If two angles are vertical angles, then they are congruent ($\cong$)



Example 4: Prove Vertical Angles are $\cong$

Given: $\angle 1$ and $\angle 2$ form a linear pair.

$\angle 2$ and $\angle 3$ form a linear pair.

Prove: $\angle 1 \cong \angle 3$

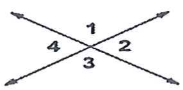
Statements	**Reasons**
1. $\angle 1$ and $\angle 2$ form a linear pair $\angle 2$ and $\angle 3$ form a linear pair	1. Given
2. $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary	2. Supplement Theorem
3. $\angle 1 \cong \angle 3$	3. $\angle$'s supplement to same $\angle$

✓ $5(19)-2 = 4(19)+17$

$95-2 = 76+17$

$93 = 93$ ✓

$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$



Example 5: Using Vertical Angles

Find the value of x using vertical angles.

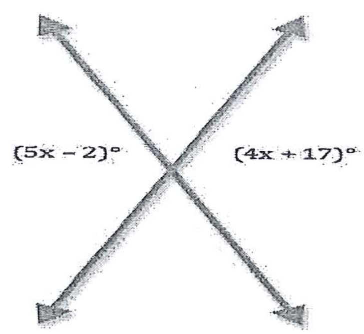
$5\cancel{x}-2 = \cancel{4x}+17$

$-4x \quad -4x$

$x-2 = 17$

$+2 \quad +2$

$x = 19$



Right $\angle$ theorems

Theorem 2.9:	Perpendicular lines intersect to four right $\angle$'s.
Theorem 2.10:	All right angles are congruent.
Theorem 2.11:	Perpendicular lines form congruent adjacent angles.
Theorem 2.12:	If two angles are congruent and supplementary, then each angle is a right $\angle$.
Theorem 2.13:	If two congruent angles form a linear pair, then they are right $\angle$'s