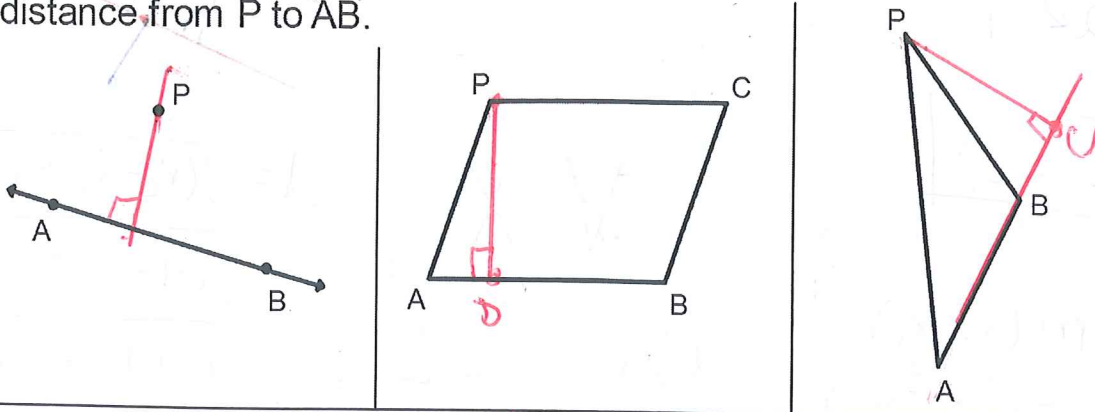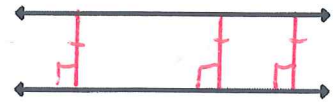# Geometry - 3.6 - Perpendiculars and Distance

- The distance from a __line__ to a point not on the line is the length of the segment __perpendicular__ to the line from the point.

Ex 1 - In the pictures below, draw a segment that represents the distance from P to $\overrightarrow{AB}$.

- Two coplanar lines are __parallel__ if they are __equidistant__ anywhere the two lines are connected with a perpendicular line.
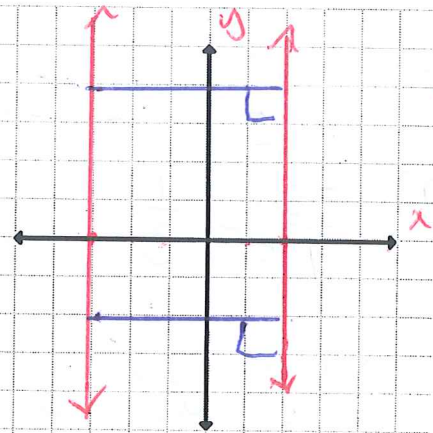

- In a plane, if two lines are __equidistant__ from a third line, then the two lines are __parallel__ to each other. (Thm. 3.9)
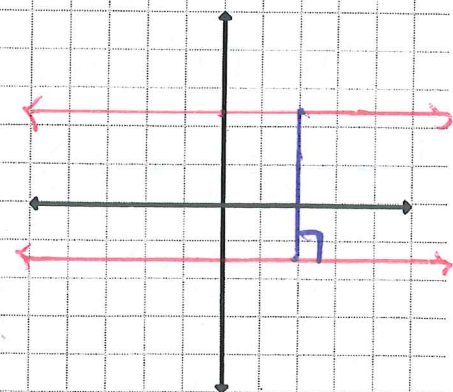

Ex 2 - Find the distance between the lines $x = -3$ and $x = 2$.

Distance $= 2 - (-3)$
$= 2 + 3$
$= 5$

Ex 3 - Find the distance between the lines $y = -1.5$ and $y = 2.3$.

$2.3 - (-1.5)$
$2.3 + 1.5$

Distance $= 3.8$

Ex 4 - Find the distance between the lines
$y = \frac{1}{2}x - 2$ and $y = \frac{1}{2}x + \frac{1}{2}$.

Slope: $\frac{1}{2} \frac{5 - 2}{2 \quad 1}$

Point $(0, -2)$
   $x_1 \quad y_1$

$m = -\frac{2}{1}$

$y - y_1 = m(x - x_1)$
$y - (-2) = -2(x - 0)$
$y + 2 = -2x$
$\quad -2 \qquad -2$
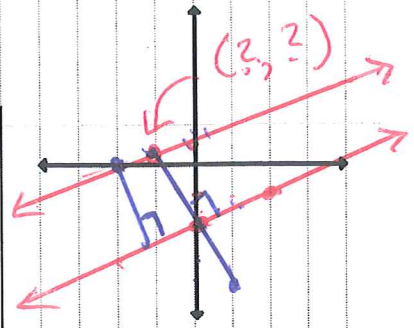$y = -2x - 2$

$y = \frac{1}{2}x + \frac{1}{2}$

$y = -2x - 2$

$\frac{1}{2}x + \frac{1}{2} = -2x - 2$
$+2x \qquad +2x$

$2\frac{1}{2}x = -2\frac{1}{2}$
$\overline{2\frac{1}{2}} \qquad \overline{2\frac{1}{2}}$

$x = -1$

$y = -2(-1) - 2$
$y = 2 - 2$
$y = 0$

$(-1, 0)$
$x_2 \quad y_2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-1 - 0)^2 + (0 + 2)^2}$
$\sqrt{(-1)^2 + (2)^2}$
$\sqrt{1 + 4}$
$\sqrt{5} = 2.23$

Ex 5 - Find the distance between the lines
$y = -3x - 3$ and $y = -3x + \frac{3}{2}$.

$m = -\frac{3}{1} \quad \frac{1}{2} = \frac{1}{3}$

Point $(0, -3)$ $\frac{1}{3}$
   $x_1 \quad y_1$

$y - y_1 = m(x - x_1)$
$y - -3 = \frac{1}{3}(x - 0)$
$y + 3 = \frac{1}{3}(x)$
$\qquad -3$
$y = \frac{1}{3}x - 3$

$y = -3x + \frac{3}{2}$
$y = \frac{1}{3}x - 3$

$-3x + \frac{3}{2} = \frac{1}{3}x - 3$
$-\frac{9}{3}x + \frac{3}{2} = \frac{1}{3}x$
$-\frac{1}{3}x \quad \frac{3}{2} \quad -\frac{1}{3}x$
$-\frac{3}{10}x = -\frac{9}{2}$
$\left(-\frac{10}{3}\right)x = -\frac{9}{2}\left(-\frac{3}{10}\right)$

$x = \frac{27}{20}$
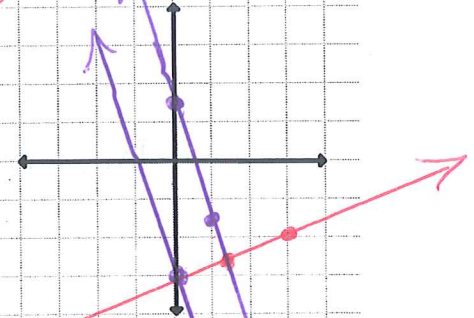
$(1.35, .25?)$

$\left(\frac{27}{20}, \frac{-51}{20}\right)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(1.35 - 0)^2 + (-2.25 + 3)^2}$
$= \sqrt{1.35^2 + (5.25)^2}$
$\sqrt{1.8 + 27.5} = \sqrt{29.3}$
$= 5.4?$

$y = -3\left(\frac{27}{20}\right) + \frac{3}{2}$
$y = \frac{-81}{20} + \frac{3}{2} \times \frac{10}{10}$
$y = \frac{-81}{20} + \frac{30}{20} = \frac{-51}{20}$