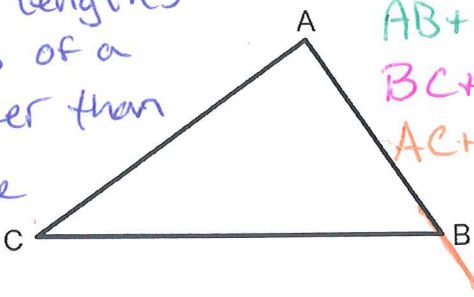


Geometry - 5.4 - The Triangle Inequality

Triangle Inequality Theorem (5.11)

The sum of 2 lengths of any 2 sides of a triangle is greater than the length of the third side.



$AB + BC > AC$
 $BC + AC > AB$
 $AC + AB > BC$

Ex 1 - Determine whether the given measures can be the lengths of the sides of a triangle.

a) 3, 4, and 6

$$\begin{aligned} 3 + 4 &> 6 \\ 7 &> 6 \checkmark \\ 4 + 6 &> 3 \\ 10 &> 3 \checkmark \\ 6 + 3 &> 4 \\ 9 &> 4 \checkmark \end{aligned}$$

b) 50, 90, and 40

$$\begin{aligned} 50 + 90 &> 40 \\ 140 &> 40 \checkmark \\ 90 + 40 &> 50 \\ 130 &> 50 \checkmark \\ 50 + 40 &> 90 \\ 90 &> 90 \text{ X} \end{aligned}$$

c) 67, 32, and 100

$$\begin{aligned} 67 + 32 &> 100 \\ 99 &> 100 \text{ X} \end{aligned}$$

Ex 2 - Find the range for the measure of the third side of a triangle given the measures of two sides.

a) 9 and 16

$$\begin{aligned} 9 + 16 &> X \\ 25 &> X \\ X &< 25 \end{aligned}$$

$$\begin{aligned} X + 9 &> 16 \\ X &> 7 \end{aligned}$$

$$7 < X < 25$$

b) 20 and 30

$$\begin{aligned} 20 + 30 &> X \\ 50 &> X \\ X &< 50 \end{aligned}$$

$$\begin{aligned} X + 20 &> 30 \\ -20 & \quad -20 \\ X &> 10 \end{aligned}$$

$$10 < X < 50$$

c) 1.8 and 3.21

$$\begin{aligned} 1.8 + 3.21 &> X \\ 5.01 &> X \\ X &< 5.01 \end{aligned}$$

$$\begin{aligned} X + 1.8 &> 3.21 \\ -1.8 & \quad -1.8 \\ X &> 1.41 \end{aligned}$$

$$1.41 < X < 5.01$$

Ex 3 - Without graphing, determine whether the given coordinates can be the vertices of a triangle.

a) D(5, 8), E(2, -4), and N(-3, -1)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$DE = \sqrt{(2-5)^2 + (-4-8)^2}$$

$$= \sqrt{(-3)^2 + (-12)^2}$$

$$= \sqrt{9 + 144}$$

$$= \sqrt{153}$$

$$= 12.4$$

$$= \sqrt{(-3-2)^2 + (-1+4)^2}$$

$$= \sqrt{(-5)^2 + (3)^2}$$

$$= \sqrt{25 + 9}$$

$$= \sqrt{34}$$

$$= 5.83$$

$$= \sqrt{(-3-5)^2 + (-1-8)^2}$$

$$= \sqrt{(-8)^2 + (-9)^2}$$

$$= \sqrt{64 + 81}$$

$$= \sqrt{145}$$

$$= 12.04$$

yes

b) A(1, -4), T(-3, -20), and L(5, 12)

$$= \sqrt{(-3-1)^2 + (-20-1)^2}$$

$$= \sqrt{(-4)^2 + (-21)^2}$$

$$= \sqrt{16 + 442}$$

$$= \sqrt{458}$$

$$\frac{32}{408}$$

$$= \sqrt{(5+3)^2 + (12+20)^2}$$

$$= \sqrt{8^2}$$

$$= \sqrt{64 + (32)^2}$$

$$= \sqrt{64 + 1024}$$

$$= \sqrt{1088}$$

$$= 33$$