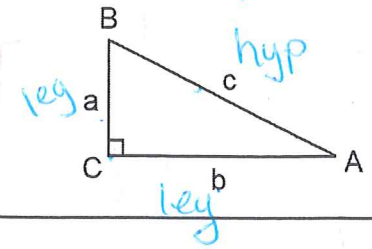
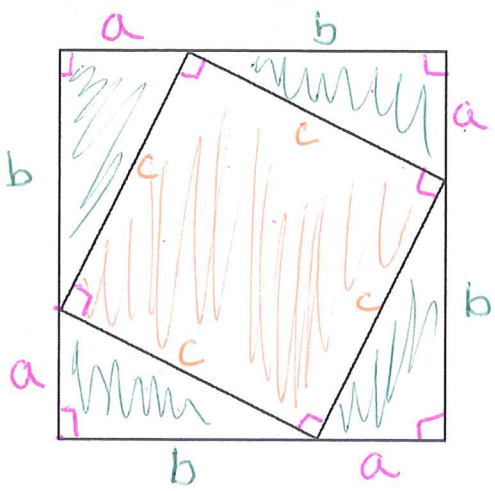
Geometry - 7.2 - The Pythagorean Theorem and Its Converse

Pythagorean Thm. (7.4)



right $\triangle$ $\longrightarrow$ $leg^2 + leg^2 = hyp^2$

$a^2 + b^2 = c^2$



One Proof of the Pythagorean Theorem

$A\triangle = \frac{1}{2} \cdot b \cdot h$

$A\square = s^2$

(4 right $\triangle$'s) + (small square) = (big square)

$4 \cdot \frac{1}{2} \cdot a \cdot b + c^2 = (a+b)^2$

$2ab + c^2 = (a+b)(a+b)$

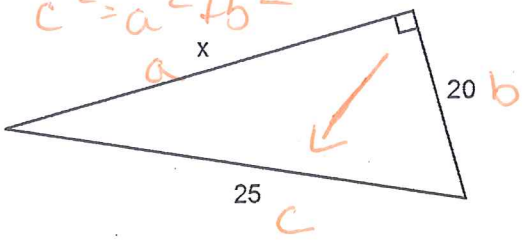
$2ab + c^2 = a^2 + ab + ab + b^2$

$2ab + c^2 = a^2 + 2ab + b^2$

$-2ab \quad\quad -2ab$

$c^2 = a^2 + b^2$

Ex 1 - Find x.



$a^2 + b^2 = c^2$

$x^2 + 20^2 = 25^2$

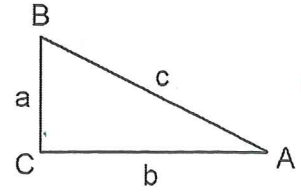
$x^2 + 400 = 625$

$-400 \quad\quad -400$

$\sqrt{x^2} = \sqrt{225}$

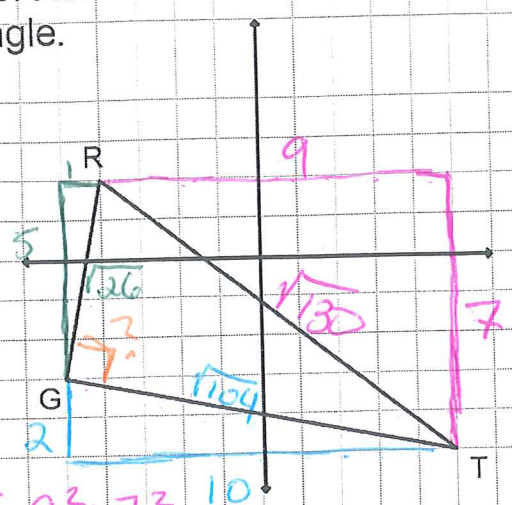
$x = 15$

Converse of Pythagorean Thm



$a^2 + b^2 = c^2$ $\longrightarrow$ right $\triangle$

Ex 2 - Using the Converse of the Pythagorean Theorem, verify that ΔRGT is a right triangle.



$5^2 + 1^2 = RG^2$

$25 + 1 = RG^2$

$\sqrt{26} = \sqrt{RG^2}$

$\sqrt{26} = RG$

$2^2 + 10^2 = GT^2$

$4 + 100 = GT^2$

$\sqrt{104} = \sqrt{GT^2}$

$\sqrt{104} = GT$

$RT^2 = 9^2 + 7^2$

$RT^2 = 81 + 49$

$\sqrt{RT^2} = \sqrt{130}$

$RT = \sqrt{130}$

$a^2 + b^2 = c^2$

$\sqrt{26}^2 + \sqrt{104}^2 = \sqrt{130}^2$

$26 + 104 = 130$

$130 = 130$ ✓

- A Pythagorean Triple is a set of three whole numbers that satisfy the Pythagorean Theorem.

Ex 3 - Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple.

a) 5, 12, 13	b) 10, 15, $\sqrt{20}$	c) 3, $\frac{3\sqrt{3}}{2}$, $\frac{3}{2}$
a b c $5^2 + 12^2 = 13^2$ $25 + 144 = 169$ $169 = 169$ ✓ Right Δ Pythag. Triple	b c a $a^2 + b^2 = c^2$ $\sqrt{20}^2 + 10^2 = 15^2$ $20 + 100 = 225$ $120 \neq 225$ NOT RIGHT NOT a Pythag Triple	c 2.10 1.5 $\left(\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 = 3^2$ $\frac{9}{4} + \frac{9 \cdot 3}{4} = 9$ $\frac{9}{4} + \frac{27}{4} = 9$ $\frac{36}{4} = 9$ $9 = 9$ ✓ right Δ Not Pythag. Triple