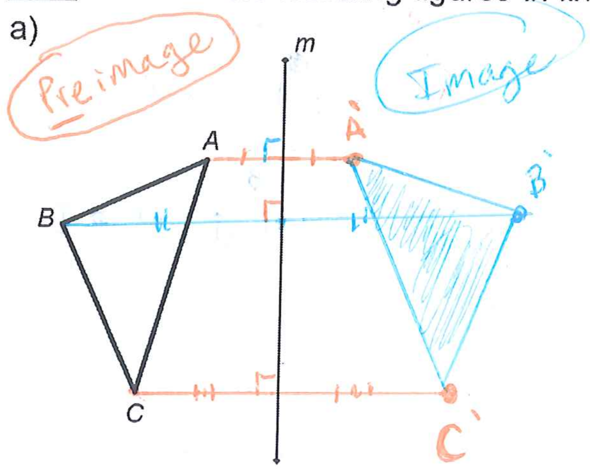
Geometry - 9.1 - Reflections

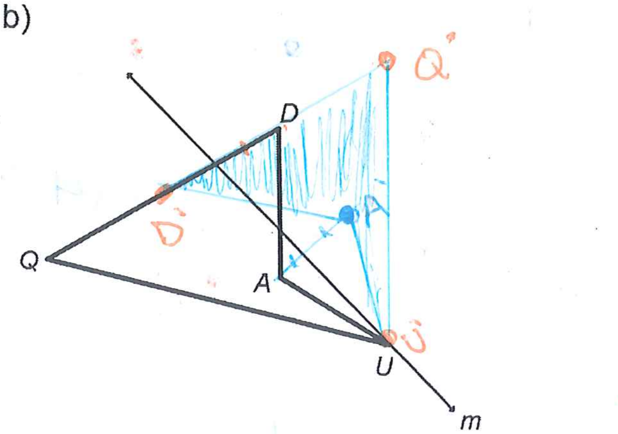
- A <u>Reflection</u> is a transformation representing the <u>flip</u> of a figure. Figures may be reflected in a <u>point</u>, <u>line</u>, or <u>plane</u>.
- An <u>isometry</u> is a transformation that preserves <u>distance</u>, <u>angle</u> <u>measures</u>, <u>betweeness</u> of points, and <u>collinearity</u>. The three types of isometries we will discuss in this chapter are <u>reflections</u>, <u>tranlations</u>, and <u>rotations</u>.

Ex 1 - Reflect the following figures in line *m*.

a)

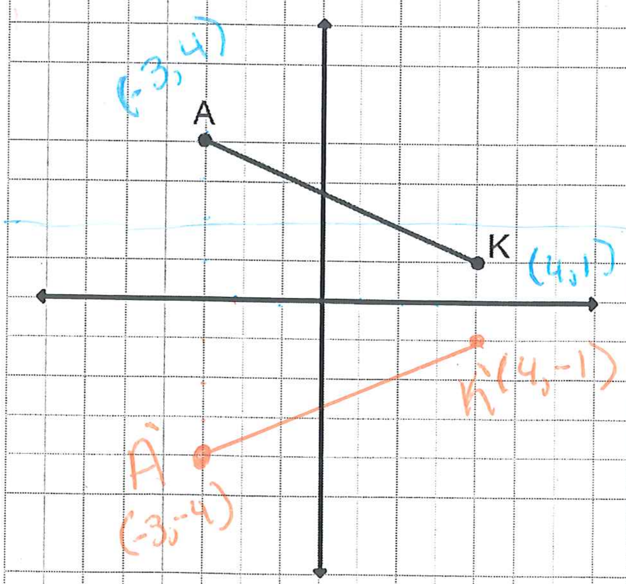


b)



Ex 2 - Reflect the following figures in specified manners:

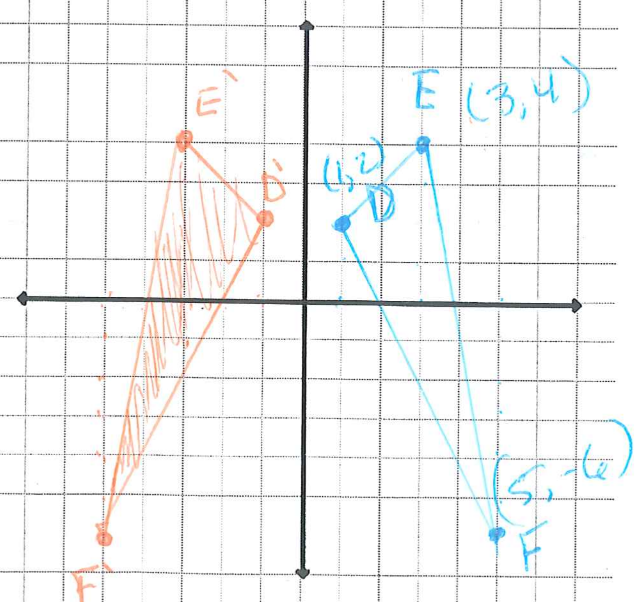
a) The given segment in the *x*-axis.



Rule for reflections in the *x*-axis:

$(a, b) \rightarrow$ <u>(a, -b)</u>

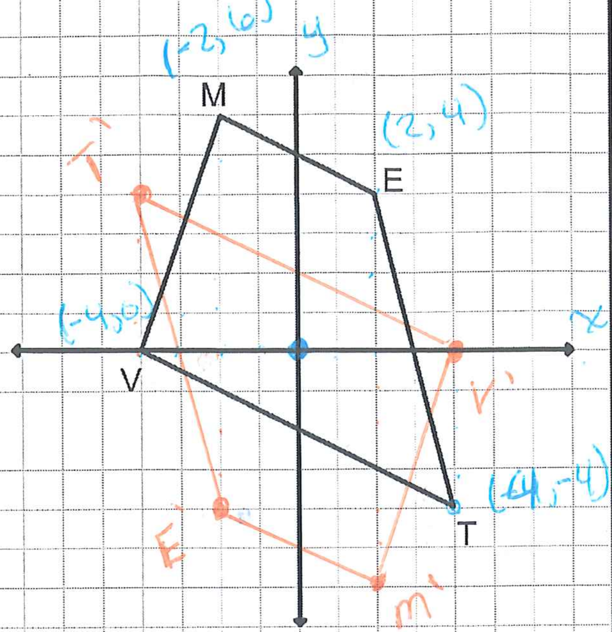
b) A triangle with coordinates $D(1, 2)$, $E(3, 4)$, and $F(5, -6)$ in the *y*-axis.



Rule for reflections in the *y*-axis:

$(a, b) \rightarrow$ <u>(-a, b)</u>

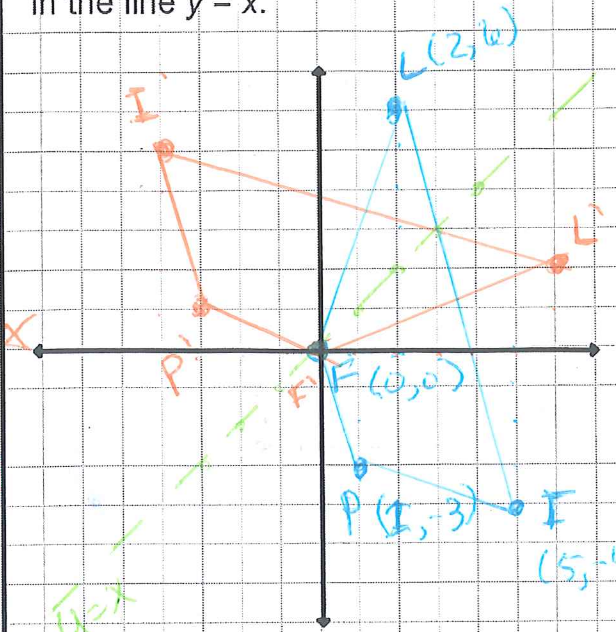
c) The given quadrilateral the origin.



Rule for reflections in the origin:

$(a, b) \rightarrow$ (-a, -b)

d) A quadrilateral with coordinates $F(0, 0)$, $L(2, 6)$, $I(5, -4)$ and $P(1, -3)$ in the line $y = x$.



Rule for reflections in the line $y = x$:

$(a, b) \rightarrow$ (b, a)

(x, y)

slope y-int

y=mx+b

m = 1/1

b = 0

y-int

y=x

F (0,0)

F'(0,0)

L (2,6)

L'(6,2)

I (5,-4)

I'(-4,5)

P (1,-3)

P'(-3,1)

- A line of symmetry is a line that can be drawn through a figure so that the figure on one side is the reflection image of the figure on the opposite side.
- A Point of symmetry is a common point of reflection for all points on a figure.

Ex 3 - Draw any lines of symmetry and points of symmetry (using a point P) on the logos below. (Ignore any shading or coloring in the logos.)

P

(No line)

(No Point)

Geometry - 9.2 - Translations

- A <u>Translation</u> is a transformation that moves all points of a figure the same <u>distance</u> in the same <u>direction</u>.

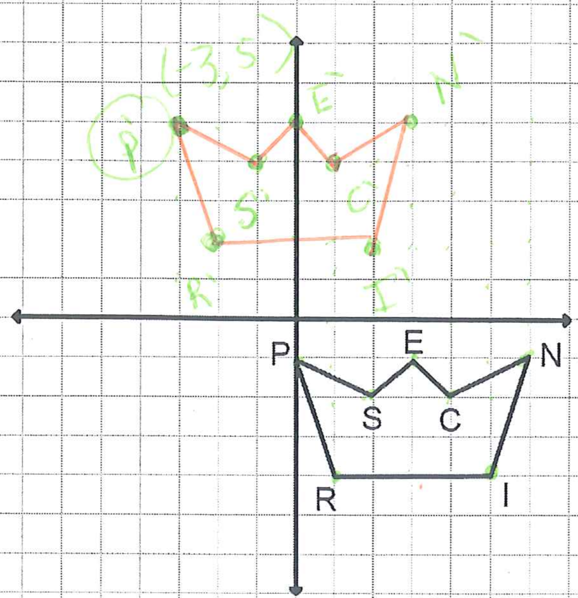
Ex 1 - Translate the figures below in the specified manners:

a) 6 units up and 3 units left

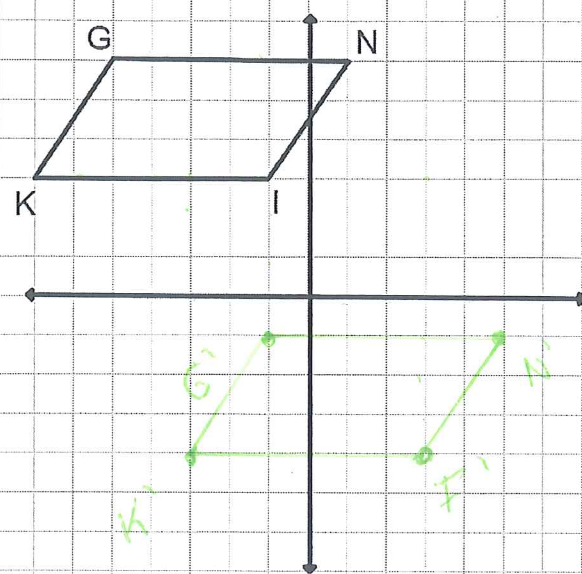
P(0,-1)

(x-3, y+6)

P'(-3, 5)

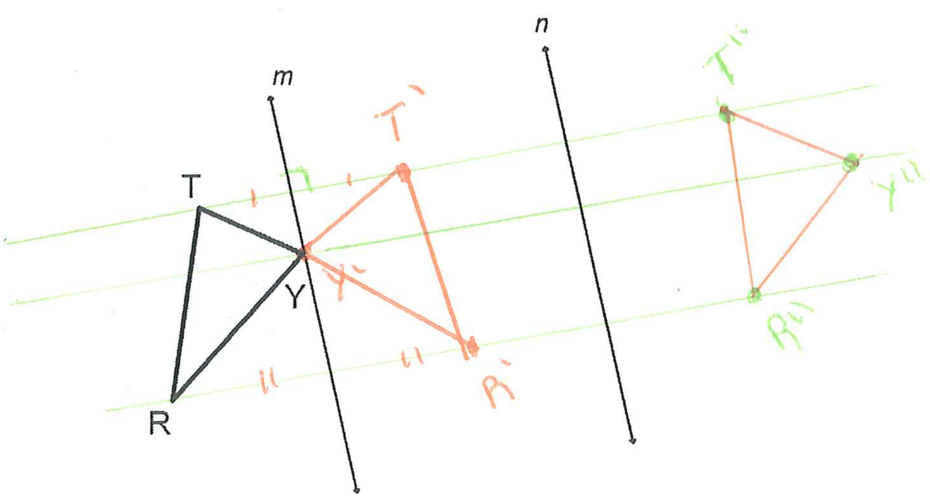


b) $(x, y) \rightarrow$ <u>(x+4, y-7)</u>



- Another way to find a translation is to perform a <u>reflection</u> in the first of two <u>parallel</u> lines and then reflect the image in the other parallel line.
- A transformation made up of successive transformations is called a <u>Composition</u>.

Ex 2 - Assuming lines *m* and *n* are parallel, perform a <u>translation</u> by reflecting the given figure first in line *m* and then its image in line *n*.

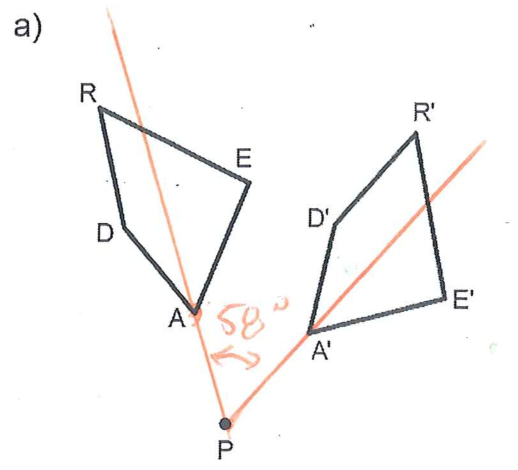


Geometry - 9.3 - Rotations

- A rotation is a transformation that turns every point of a preimage through a specified angle and direction about a fixed point. The fixed point is called the center of rotation and the angle is called the angle of rotation.

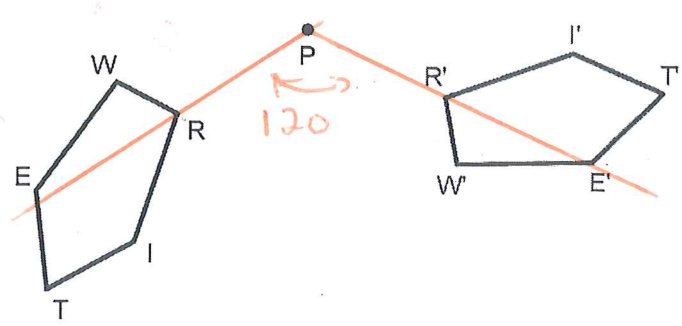
Ex 1 - Determine the angle of rotation about point P.

a)



58° Clockwise

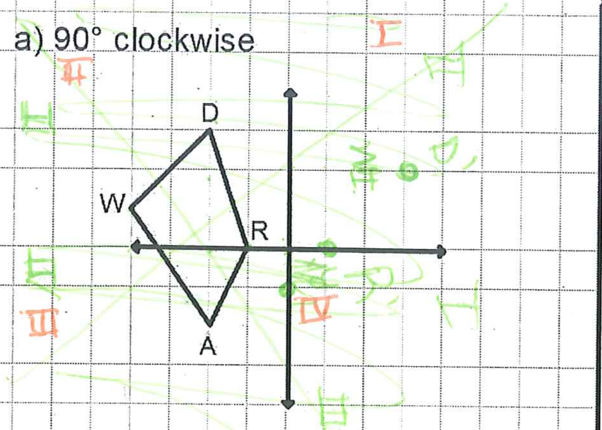
b)



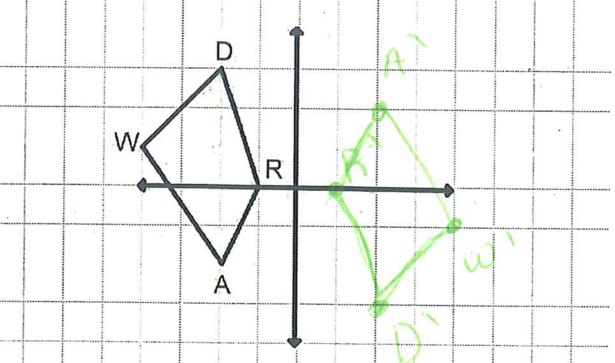
120° counter clockwise

Ex 2 - Draw the image of the figure under the specified rotation about the origin.

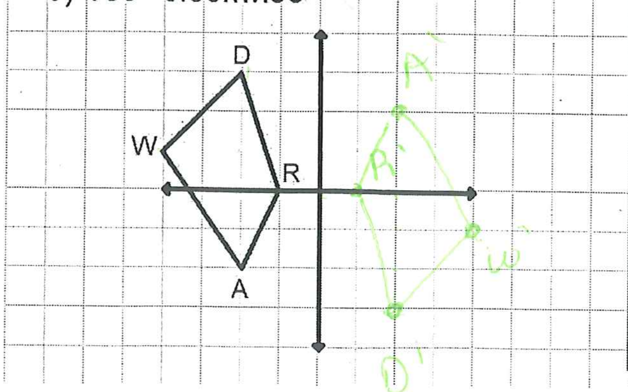
a) 90° clockwise



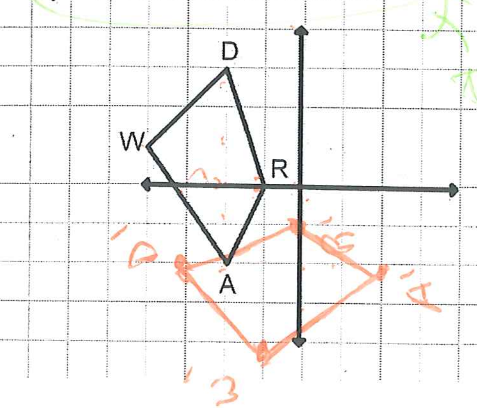
b) 180° counterclockwise



c) 180° clockwise

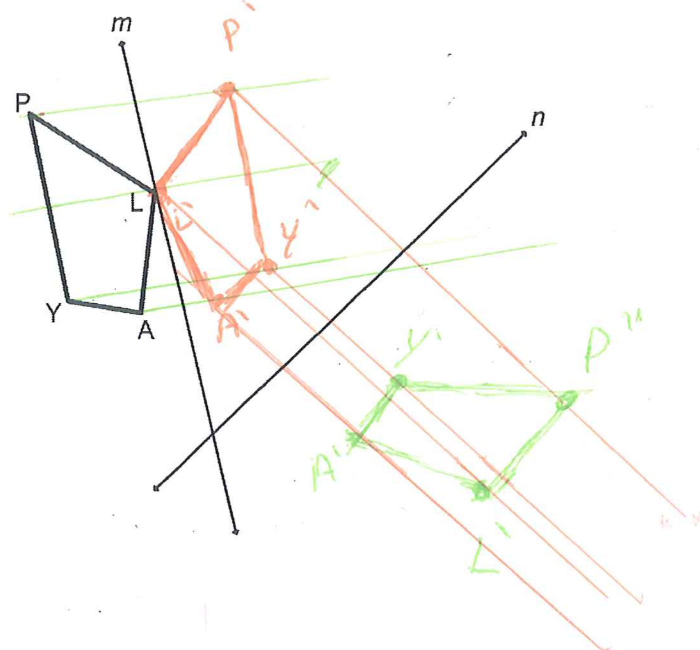


d) 90° counterclockwise



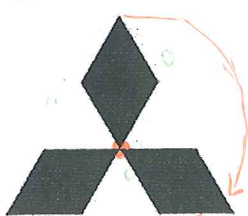
Turn the paper clockwise + redraw

Ex 3 - Assuming lines *m* and *n* are not parallel, perform a rotation by reflecting the given figure first in line *m* and then its image in line *n*.



- If a figure can be rotated less than 360 degrees about a point so that the preimage and image are indistinguishable, then the figure has rotational symmetry.

Ex 4 - If rotational symmetry exists, determine the order and magnitude of the rotational symmetry for each figure below.



Order = 3

$\frac{360}{3} = 120^\circ$

Magnitude = 120°

No rotational Symmetry

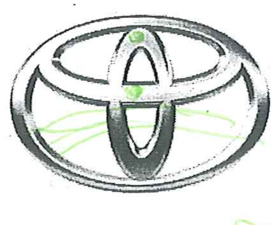
Order = 2

Magnitude = $\frac{360^\circ}{2}$

180°

Order = 4

Magnitude 90°



No rotational Symmetry

Order = 6

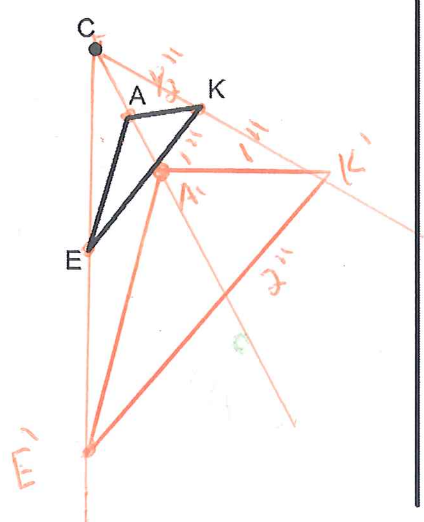
magnitude = $\frac{360}{6} = 60^\circ$

Geometry - 9.5 - Dilations

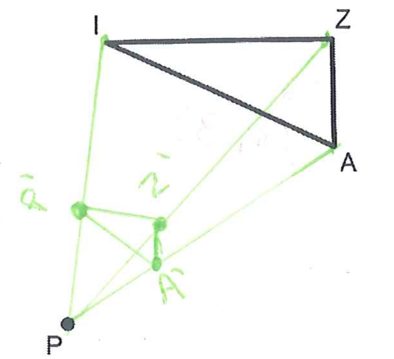
- A dialation is a similarity transformation (not an isometry) that may change the size of a figure. A dilation requires a center point and a scale factor, which we label as r.

Ex 1 - Draw the dilation image for each figure below.

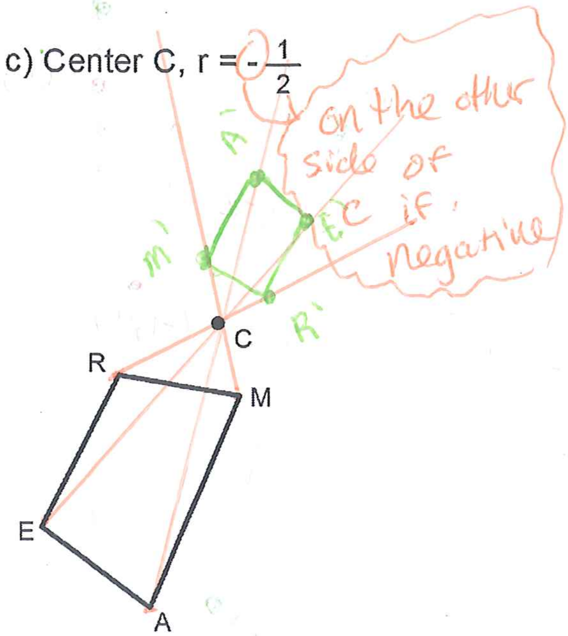
a) Center C, r = 2



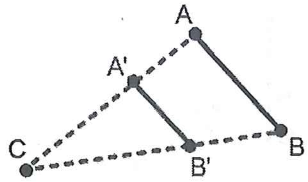
b) Center P, $r = \frac{1}{3}$



c) Center C, $r = -\frac{1}{2}$



- If $|r| > 1$, a dilation is an enlargement.
  If $0 < |r| < 1$, a dilation is a reduction.
  If $r = 1$, a dilation is a congruent transformation.



- If a dilation with center *C* and scale factor *r* transforms *A* to *A'* and *B* to *B'*, then $A'B' = |r| \cdot AB$.

Ex 2 - Find the measure of the dilation image $\overline{A'B'}$ or the preimage $\overline{AB}$ using the given scale factor.

a) AB = 13, r = -3

$A'B' = |r| \cdot AB$

$A'B' = |-3| \cdot 13$

$A'B' = 3 \cdot 13$

$A'B' = 39$

b) A'B' = 45, $r = \frac{1}{5}$

$A'B' = |r| \cdot AB$

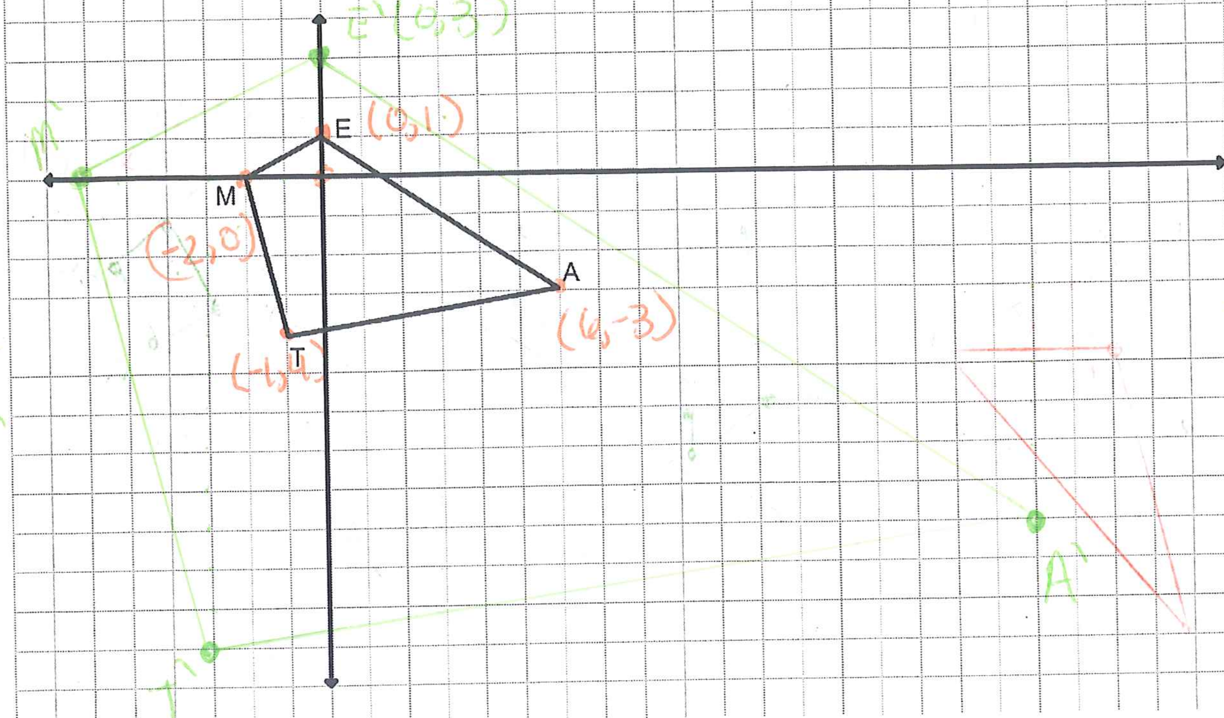
$45 = |\frac{1}{5}| \cdot AB$ $\quad \frac{1}{5} \times \frac{5}{1}$

$\frac{5}{1} \cdot 45 = \frac{1}{5} \cdot AB \cdot \frac{5}{1}$

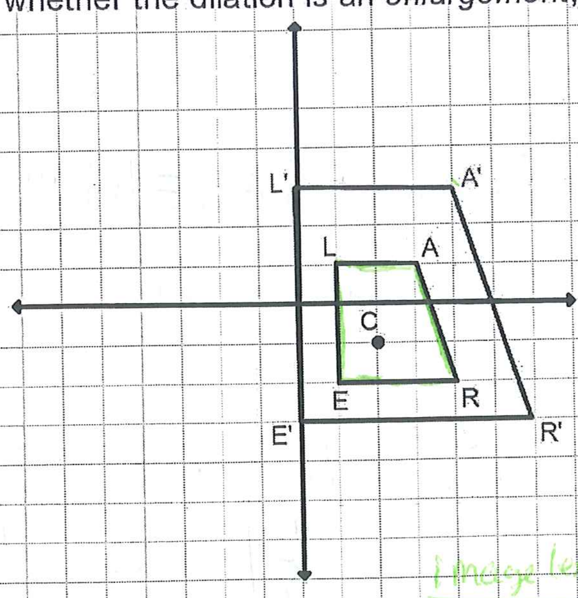
$225 = AB$

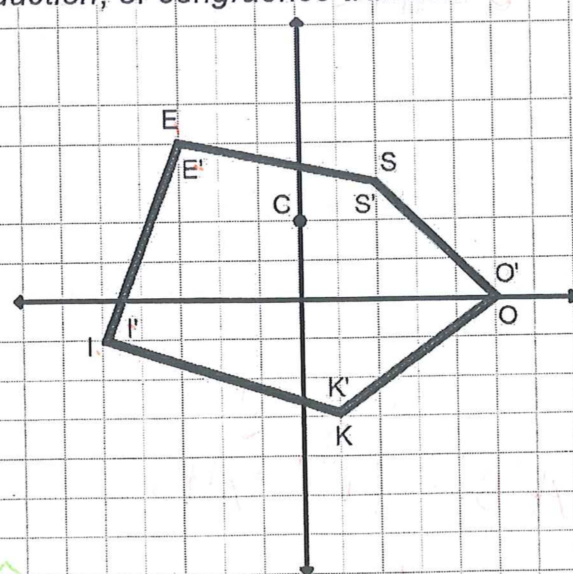
- If P(x, y) is the preimage of a dilation centered at the origin with a scale factor r, then the image is P(rx, ry).

Ex 3 - Draw the image of the quadrilateral below after a dilation centered at the origin with a scale factor of 3.



Ex 4 - Determine the scale factor for each dilation with center C. Then determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.





Scale Factor = $\frac{\text{Image length}}{\text{Preimage length}}$

$\frac{A'L'}{AL} = \frac{4}{2} = 2$

Enlargement

Congruence Transformation