

Simplifying Radical Expressions--Notes

Name: Key Date: _____ Hour: _____

Radical expressions contain a radical.

radical \swarrow
 $\sqrt{2} + 3$

The radicand is the quantity or expression under the radical sign.

radicand \swarrow
 $\sqrt{x-6}$

$\sqrt{0} = 0$ $\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$ $\sqrt{16} = 4$ $\sqrt{25} = 5$

$\sqrt{36} = 6$ $\sqrt{49} = 7$ $\sqrt{64} = 8$ $\sqrt{81} = 9$ $\sqrt{100} = 10$ $\sqrt{121} = 11$

The numbers inside the radicals in the preceding two rows are examples of radical expression because...

Examples: Simplify the Radical Expression by Removing Perfect-Square Factors

a) $\sqrt{27}$

$\sqrt{9 \cdot 3}$
 $3\sqrt{3}$

b) $4\sqrt{200}$

$4 \cdot \sqrt{100 \cdot 2}$
 $4 \cdot 10 \cdot \sqrt{2}$
 $40\sqrt{2}$

c) $2\sqrt{45}$

$2 \cdot \sqrt{9 \cdot 5}$
 $2 \cdot 3 \cdot \sqrt{5}$
 $6\sqrt{5}$

d) $2\sqrt{32}$

$2 \cdot \sqrt{16 \cdot 2}$
 $2 \cdot 4 \cdot \sqrt{2}$
 $8\sqrt{2}$

e) $\sqrt{75}$

$\sqrt{25 \cdot 3}$
 $5\sqrt{3}$

f) $\sqrt{72}$

$\sqrt{36 \cdot 2}$
 $6\sqrt{2}$

Rationalizing a Denominator

Multiply top and bottom by the radical

a) $\frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2\sqrt{4}}$

$\frac{3\sqrt{2}}{2 \cdot 2}$

$\frac{3\sqrt{2}}{4}$

b) $\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{\sqrt{25}}$

$\frac{4\sqrt{5}}{5}$

c) $\frac{6}{\sqrt{12}} = \frac{6}{\sqrt{4 \cdot 3}} = \frac{6}{2\sqrt{3}}$

$\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

d) $\frac{10}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{10\sqrt{3}}{\sqrt{9}}$

$\frac{10\sqrt{3}}{3}$

e) $\frac{5}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{5\sqrt{2}}{4 \cdot \sqrt{4}}$

$\frac{5\sqrt{2}}{4 \cdot 2} = \frac{5\sqrt{2}}{8}$

f) $\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{3 \cdot 2}}{\sqrt{4}}$

$\frac{\sqrt{6}}{2}$

$\frac{3\sqrt{3}}{\sqrt{9}}$

$\frac{3\sqrt{3}}{3\sqrt{3}}$

Multiplying and Squaring Radicals

Multiply "numbers with numbers" and "radicals with radicals". Then simplify further if needed.

a) $2\sqrt{5} \cdot 3\sqrt{8}$
 $6\sqrt{40}$
 $6 \cdot \sqrt{4} \cdot \sqrt{10}$
 $6 \cdot 2 \cdot \sqrt{10}$

$12\sqrt{10}$

b) $\sqrt{10} \cdot 4\sqrt{5}$
 $4\sqrt{50}$
 $4 \cdot \sqrt{25} \cdot \sqrt{2}$
 $4 \cdot 5 \cdot \sqrt{2}$
 $20\sqrt{2}$

c) $6\sqrt{6} \cdot 4\sqrt{2}$
 $24\sqrt{12}$
 $24 \cdot \sqrt{4} \cdot \sqrt{3}$
 $24 \cdot 2 \cdot \sqrt{3} = 48\sqrt{3}$

(Simplify the following--you could rewrite these first, but you don't always need to....)

d) $(\sqrt{8})^2$
 $\sqrt{8} \cdot \sqrt{8}$
 $\sqrt{64}$
 8

e) $(2\sqrt{5})^2$
 $2\sqrt{5} \cdot 2\sqrt{5}$
 $4\sqrt{25}$
 $4 \cdot 5$
 20

f) $10(\sqrt{3})^2$
 $10 \cdot 3$
 30
 $10 \cdot \sqrt{3} \cdot \sqrt{3}$
 $10 \cdot \sqrt{9}$
 $10 \cdot 3 \rightarrow 30$

g) $(\sqrt{811,367})^2$
 $811,367$

Simplifying Radicals by Dividing

Divide radicals with radicals and numbers with numbers (if they are divisible). Then simplify further if needed.

a) $\frac{\sqrt{120}}{\sqrt{6}} = \sqrt{\frac{120}{6}} = \sqrt{20}$
 $\sqrt{4} \cdot \sqrt{5}$
 $2 \cdot \sqrt{5}$

b) $\frac{\sqrt{200}}{\sqrt{2}} = \sqrt{\frac{200}{2}}$
 $\sqrt{100}$
 10

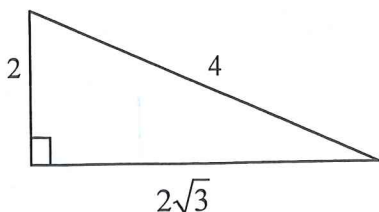
c) $\frac{10\sqrt{6}}{5\sqrt{2}} = \frac{10}{5} \cdot \frac{\sqrt{6}}{\sqrt{2}}$
 $2 \cdot \sqrt{3}$

Summary: A radical expression is in simplest radical form when these statements are true:

- The radicand has no perfect-square factors other than 1.
- The denominator of a fraction has no radical.
- Fractions are reduced.
- Like terms have been added and/or subtracted.
- Simplifying is NOT rounding our answer with a decimal! That is called approximating. *Simplifying involves an EXACT answer. Approximating involves a ROUNDED answer.*

Example: Give the last answer (c) again, but rounded to the nearest tenth. 3.5

Geometry Example: write the ratio of



a) the shortest leg to the *hypotenuse*

$\frac{2}{4} = \frac{1}{2}$

b) the *hypotenuse* to the *longest* leg.

$\frac{4}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$

Be sure to simplify any fractions or radicals! ☺